| Course Category | TBA | Credits | 2 |
|---|---|---|---|
| Subject Code | TBA | Taking Year | 1st Grade, 2nd Grade |
| Course Title (Japanese) | 力学系とエルゴード理論 | Course Period | 2nd Semester |
| Course Title | Dynamical Systems and Ergodic Theory | Day of the week / Hour | Friday / The third period |
| Registration Code | TBA | Compulsory / Elective | Elective |
| Instructor(s) | Johannes Jaerisch | Course Qualification | Students of Postgraduate Mathematics Course |

| Course Style | Lecture |
|---|---|
| Course Aim | Dynamical systems is the study of the long-term behavior of systems whose state evolves with time. We focus on chaotic dynamical systems which are highly sensitive to the initial conditions originating from celestial mechanics at the end of the 19th century. The ergodic theory studies the average behavior of a dynamical system with respect to a measure. |
| Goals and Objectives (Level of Achievement) | This is an introductory course on topological and measure-theoretic aspects (that is, ergodic theory) of chaotic dynamical systems. Topics in topological dynamics include chaotic dynamical systems (e.g., expanding maps, hyperbolic toral automorphisms, shift spaces), complexity and periodic points, points with dense orbits, minimality and almost-periodicity. Topics in ergodic theory include the concept of measure preserving transformations, ergodicity, character theory, Birkhoff's ergodic theorem and induced transformations. |
| Course Plan | 1. Circle rotations<br>2. Expanding maps on the circle<br>3. Hyperbolic toral automorphisms<br>4. Symbolic dynamics of topological Markov chains<br>5. Perron-Frobenius theorem<br>6. Counting periodic points and the dynamical zeta function<br>7. Minimality and topological transitivity<br>8. Almost periodicity<br>9. Measure-preserving dynamical systems<br>10. Poincaré-recurrence theorem<br>11. Characterizations of ergodicity<br>12. Character theory<br>13. Birkhoff's Ergodic theorem and its proof<br>14. Birkhoff's Ergodic theorem and its applications<br>15. Induced transformations and Kakutani's skyscrapers<br>16. Summary and outlook |
| Teaching Methods | There will be homework assignments in class. |
| Key Words | Dynamical systems, Ergodic theory, Chaos, Butterfly effect, Markov chains. |
| Texts | [1] Brin, M., Stuck, G.: Introduction to Dynamical Systems, Cambridge Univ. Press, 2002.<br>[2] Walters, P.: An introduction to Ergodic Theory, vol. 79, New York, Springer, 1982.<br>[3] Hasselblatt, B., Katok, A.: A First Course in Dynamics - with a Panorama of Recent Developments, 2003. |
| Reference Books | [1] Petersen, K., Ergodic theory, Cambridge Studies in Advanced Mathematics, 2. Cambridge University Press, Cambridge, 1983.<br>[2] Barreira, Luis and Valls Claudia. Dynamical systems. Universitext, 2013. |
| Other Teaching Materials | Further references and materials will be given in class. |
| Performance Evaluation | Evaluation is based up on homework assignments and class attendance. |
| Notes on the Course | It is strongly recommended to work on the homework assignments. |
| Office Hour | TBA |
| Other Notes | None |